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Vortices-induced quantum Röntgen effect in BEC: a consistent approach

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Abstract

Through the application of ϕ-mapping topological theory, the properties of vortices in quantum Röntgen effect are thoroughly studied. The explicit expression of the vorticity is obtained, in which the δ function indicates that the vortices can only stem from the zero points of ϕ, and the magnetic flux of the consequent monopoles is quantized in terms of the Hopf indices and Brouwer degrees. The evolution of vortex lines is discussed. The reduced dynamic equation and a conserved dynamic quantity on stable vortex lines are obtained.

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1. Introduction

Intensive studies, both theoretical and experimental, have been carried out since the advent of Bose–Einstein condensate [1, 2]. Because of its unique features, novel methods of investigation have been springing up in recent decades. By virtue of recent theory on quantum phase of induced dipoles [3–6], Leonhardt and Piwnicki studied quantum Röntgen effect via a mean-field approach (the Gross–Pitaevskii theory) and obtained some intriguing results on its quantized monopoles [7].

Given a charged capacitor and a dielectric (such as glass, robber, etc) disc placed parallel between the plates, the disc would easily be polarized, and, while it rotates, the induced charges on its opposite surfaces would act as currents and then generate a magnetic field. This effect, which is known as Röntgen effect, was discovered by W C Röntgen in 1888 [8]. Recent interests in Röntgen effect have taken place due to its promising perspective in the research of quantum gases. For example, if the ordinary dielectric disc be replaced by a quantum dielectric disc (a Bose–Einstein condensate), Leonhardt and Piwnicki showed in their paper that only vortices would generate a magnetic field and the field would behave as it originates from a set of magnetic monopoles [8]. Despite their ingenuity, they employed the traditional way to express the wavefunction

$$\psi = |\psi|\,\mathrm{e}^{\mathrm{i}S}, \tag{1}$$

which inevitably leads to the irrotational result

$$\nabla \times \vec{u} = \nabla \times \left(\frac{\hbar}{m}\nabla S\right) = 0, \tag{2}$$

and causes an irreconcilable flaw to the whole theory.

Through years of study, we have formulated a systematic approach to solve such problems—we call it ϕ-mapping topological theory. By describing the physical system in question with another vector field $\vec{\phi}$, we can explicitly show its topological structures and topological invariants, especially the δ functional behavior of the singularities. Basic details can be found in [9]; further results have been provided in various physical systems [10–14].

In this paper, through the application of ϕ-mapping topological theory, we will not only solve the problem that exists in Leonhardt and Piwnicki's work, but will also provide a thorough study on the behavior of vortices in quantum Röntgen effect. The results we obtain here would be very conducive to further theoretical and experimental investigation of BEC.

This paper is organized as follows: in section 2, we first elucidate that the curl of the velocity field is

$$\nabla \times \vec{u} = \frac{h}{m}\vec{J}\left(\frac{\phi}{x}\right)\delta^2(\vec{\phi}), \tag{3}$$

which indicates that the magnetic monopoles are generated from the zero points of ψ (i.e. the locations of vortices). While those zero points are regular points, the magnetic flux of the monopoles is quantized in terms of the Hopf indices β_i and Brouwer degrees η_i of ϕ-mapping.

In section 3, we study the critical points of ψ , i.e., the limit points and bifurcation points, and show the existence of branch processes. The vortex lines generate, annihilate, split or merge at such points, while their topological number—winding number—$\beta\eta$ is conserved.

In section 4, we investigate the reduced dynamic equation and obtain a conserved dynamic quantity on stable vortex lines. In section 5, we draw our conclusions. SI units are used throughout the paper.

2. The quantized vorticity and magnetic flux

Our work begins with the Gross–Pitaevskii equation which one can readily obtain from the Lagrangian density of [7]

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{\mathrm{i}}\nabla + \alpha\vec{E}\times\vec{B}\right)^2\psi - \frac{\alpha}{2}E^2\psi + \frac{g}{2}|\psi|^2\psi + V\psi. \tag{4}$$

As shown by Leonhardt and Piwnicki, the constant α denotes the electrical polarizability of the condensed atoms; the Gross–Pitaevskii term $g|\psi|^2\psi\,(g > 0)$ models the atomic repulsion (collision), which tends to smooth out density variations over the healing length $\hbar/\sqrt{2gm|\psi|^2}$; and V stands for the external potential which prevents the condensate from spreading out to infinity.

Instead of adopting the traditional expression (1), we shall write

$$\psi = \phi^1 + \mathrm{i}\phi^2, \tag{5}$$

where

$$\phi^a = \phi^a(x), \qquad a = 1, 2, \tag{6}$$

are real functions. It means that ψ can be described by a two-dimensional vector field $\vec{\phi} = (\phi^1, \phi^2)$.

Substituting (5) into (4), we get the dynamic equation of the polarized condensed atoms

$$-\hbar\varepsilon_{ab}\phi^a\frac{\partial}{\partial t}\phi^b = -\frac{\hbar^2}{2m}\phi^a\nabla^2\phi^a + \frac{\hbar}{m}(\alpha\vec{E}\times\vec{B})\cdot\varepsilon_{ab}\phi^a\nabla\phi^b + \left[\frac{1}{2m}(\alpha\vec{E}\times\vec{B})^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + V\right]\|\phi\|^2, \tag{7}$$

as well as the continuity relation

$$\frac{\partial}{\partial t}\|\phi\|^2 + \nabla\cdot\left[\left(\frac{\hbar}{m}\frac{1}{\|\phi\|^2}\varepsilon_{ab}\phi^a\nabla\phi^b + \frac{\alpha}{m}\vec{E}\times\vec{B}\right)\|\phi\|^2\right] = 0, \tag{8}$$

where

$$\|\phi\|^2 = \phi^a\phi^a = \psi^*\psi \tag{9}$$

describes the density of the condensed atoms.

Consequently, the velocity field takes the form

$$\vec{u} = \frac{\hbar}{m}\frac{1}{\|\phi\|^2}\varepsilon_{ab}\phi^a\nabla\phi^b + \frac{\alpha}{m}\vec{E}\times\vec{B}. \tag{10}$$

In the following study, the second term is omitted as its contribution to the outcome is negligibly small (which was shown in [7]).

Note that

$$\frac{\phi^a}{\|\phi\|^2} = \frac{1}{\|\phi\|}\frac{\partial}{\partial\phi^a}\|\phi\| = \frac{\partial}{\partial\phi^a}\ln\|\phi\|, \tag{11}$$

curl $\vec{u}$ can be expressed as

$$\nabla\times\vec{u} = \frac{\hbar}{m}(\nabla\phi^1\times\nabla\phi^2)\frac{\partial^2}{\partial\phi^a\partial\phi^a}\ln\|\phi\|. \tag{12}$$

By defining the Jacobian vector

$$\vec{J}\left(\frac{\phi}{x}\right) = \nabla\phi^1\times\nabla\phi^2, \tag{13}$$

and utilizing the Laplacian relation in ϕ space [15]

$$\frac{\partial^2}{\partial\phi^a\partial\phi^a}\ln\|\phi\| = 2\pi\delta^2(\vec{\phi}), \tag{14}$$

(12) can be reduced to

$$\nabla\times\vec{u} = \frac{h}{m}\vec{J}\left(\frac{\phi}{x}\right)\delta^2(\vec{\phi}). \tag{15}$$

This explicit expression is the utter solution to the problem which we mentioned in section 1. Actually, such a δ-functional behavior of the vorticity had been assumed by Leonhardt and Piwnicki, but they were unable to provide any proof.

From the relation between $\vec{H}$ and $\vec{u}$ [7]

$$\nabla\cdot\vec{H} = \varepsilon_0\chi\vec{E}\cdot(\nabla\times\vec{u}), \tag{16}$$

we have

$$\nabla\cdot\vec{H} = \frac{h}{m}\varepsilon_0\chi\vec{E}\cdot\vec{J}\left(\frac{\phi}{x}\right)\delta^2(\vec{\phi}). \tag{17}$$

Therefore, one can easily reach the conclusion that $\nabla \cdot \vec{H} \neq 0$ (i.e.$\nabla \cdot \vec{u} \neq 0$) if and only if $\vec{\phi} = 0$, the magnetic monopoles are generated from the zero points of $\vec{\phi}$ (i.e. the locations of vortices). This makes the solution of $\vec{\phi} = 0$ extremely significant.

Suppose the vector field $\vec{\phi}(x)$ has N zero points $\vec{z}_i (i = i, \ldots, N)$. The implicit function theorem assures us, while $\vec{z}_i (i = i, \ldots, N)$ are the regular points of $\vec{\phi}(x)$, i.e. $\vec{J}\left(\frac{\phi}{x}\right)|_{\vec{z}_i} \neq 0$, they can be expressed as parameterized singular strings:

$$L_i : \quad \vec{z}_i = \vec{z}_i(t, s), \qquad i = 1, \ldots, N. \tag{18}$$

These N isolated singular strings are just the vortex lines.

For a fixed t, one can obtain the following result from the δ-function theory [15]

$$\delta^2(\vec{\phi}) = \sum_{i=1}^{N} \beta_i \int_{L_i} \frac{\delta^3(\vec{x} - \vec{z}_i(s))}{|J(\phi/u)|_{\Sigma_i}} \, \mathrm{d}s, \tag{19}$$

where Σ_i is the ith planer element transversal to L_i with local coordinates (u^1, u^2), $J\left(\frac{\phi}{u}\right) = \frac{\partial(\phi^1, \phi^2)}{\partial(u^1, u^2)}$, and β_i is a positive integer known as the Hopf index of ϕ-mapping.

Since the direction vector of L_i is given by

$$\left.\frac{\mathrm{d}\vec{x}}{\mathrm{d}s}\right|_{\vec{z}_i} = \left.\frac{\vec{J}(\phi/x)}{J(\phi/u)}\right|_{\vec{z}_i}, \tag{20}$$

we then have

$$\nabla \times \vec{u} = \frac{h}{m} \sum_{i=1}^{N} \beta_i \eta_i \int_{L_i} \frac{\mathrm{d}\vec{x}}{\mathrm{d}s} \delta^3(\vec{x} - \vec{z}_i(s)) \, \mathrm{d}s, \tag{21}$$

where $\eta_i = \operatorname{sgn} J\left(\frac{\phi}{u}\right) = \pm 1$ is the Brouwer degree, and it characterizes the direction of the ith vortex line. This expression shows the important inner topological structure of $\nabla \times \vec{u}$.

Thus, the vorticity of the condensate is

$$\Gamma = \int_{\Sigma} \nabla \times \vec{u} \cdot \mathrm{d}\vec{S} = \frac{h}{m} \sum_{i=1}^{N} \beta_i \eta_i, \tag{22}$$

and the magnetic flux of the consequent monopoles can be worked out as

$$\Phi = \int \mu_0 \nabla \cdot \vec{H} \mathrm{d}v = \frac{h}{mc^2} \chi U \sum_{i=1}^{N} \beta_i \eta_i = \frac{\chi}{c^2} U \Gamma. \tag{23}$$

Here U denotes the applied voltage of the capacitor, χ is the susceptibility. It is obvious that both Γ and Φ are quantized by the product $\beta\eta$, which is called the winding number.

3. The evolution of vortex lines

The discussion we have been carrying on so far is under the condition that $\vec{z}_i (i = 1, \ldots, N)$ are regular points, but what happens when some $\vec{z}^* = (t^*, \vec{x}^*)$ are critical points (i.e. $\vec{J}(\frac{\phi}{x})|_{\vec{z}^*} = 0$)? Soon we will prove that such vortex lines would no longer be stable—it would evolve. In other words, generating and annihilating of vortex lines, as well as splitting and merging, would take place.

If we still insist on using the implicit function theorem, we can assume that one of the following Jacobians

$$D^i\left(\frac{\phi}{x}\right)\bigg|_{\vec{z}^*} = \left.\frac{\partial(\phi^1, \phi^2)}{\partial(t, x^i)}\right|_{\vec{z}^*}, \qquad i = 1, 2, 3, \tag{24}$$

is nonzero.

For simplicity, let us fix the z coordinate and consider the locus of $\vec{w}^* = (t^*, x^*, y^*)$.

Case 1. At least one of (24)'s Jacobians is nonzero. (Such $\vec{z}^*$ is called the limit point of $\vec{\phi}(x)$.) Suppose $D^2\left(\frac{\phi}{x}\right)\big|_{\vec{z}^*} \neq 0$, near $\vec{w}^*$ we could locally solve $\vec{\phi} = 0$ to obtain

$$t = t(x), \qquad y = y(x). \tag{25}$$

By differentiating the identity $\vec{\phi}(x, t(x), y(x)) = 0$ with respect to x, we yield

$$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{J^3(\phi/x)}{D^2(\phi/x)}, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{D^1(\phi/x)}{D^2(\phi/x)}. \tag{26}$$

Then (25) can be expanded at $\vec{w}^*$ as

$$t = t^* + \frac{\mathrm{d}t}{\mathrm{d}x}\bigg|_{\vec{w}^*}(x - x^*) + \frac{1}{2}\frac{\mathrm{d}^2t}{\mathrm{d}x^2}\bigg|_{\vec{w}^*}(x - x^*)^2 + o(|x - x^*|^2), \tag{27}$$

i.e.,

$$t - t^* = \frac{1}{2}\frac{\mathrm{d}^2t}{\mathrm{d}x^2}\bigg|_{\vec{w}^*}(x - x^*)^2, \tag{28}$$

which is a parabola on the xt plane.

Equation (28) reveals that there exist branch processes at the limit point. If $\frac{\mathrm{d}^2t}{\mathrm{d}x^2}\big|_{\vec{w}^*} > 0$, we have the branch solutions $x_1(t)$ and $x_2(t)$ for $t > t^*$, which represent the generating process. If $\frac{\mathrm{d}^2t}{\mathrm{d}x^2}\big|_{\vec{w}^*} < 0$, we have the branch solutions $x_1'(t)$ and $x_2'(t)$ for $t < t^*$, which represent the annihilating process.

Besides, equation (28) can also show a simple approximate relation near $\vec{w}^*$

$$|x - x^*| \propto |t - t^*|^{\frac{1}{2}}, \tag{29}$$

from which we can obtain the generating/annihilating speed:

$$v \propto |t - t^*|^{-\frac{1}{2}}. \tag{30}$$

No mater which process takes place, from the identity $\nabla \cdot (\nabla \times \vec{u}) = 0$, one can always reach the conclusion that the winding number $\beta\eta$ is conserved

$$\beta_1\eta_1 + \beta_2\eta_2 = 0, \tag{31}$$

which indicates that the two vortex lines have the same Hopf index and opposite directions.

Case 2. All of (24)'s Jacobians are zero. (Such $\vec{z}^*$ is called the bifurcation point of $\vec{\phi}(x)$.) Suppose one of the partial derivatives is nonzero. Let, for example, $\frac{\partial\phi^1}{\partial y} \neq 0$. Again, according to the implicit function theorem, near $\vec{w}^*$ we could locally solve $\phi^1 = 0$ to obtain

$$y = y(t, x). \tag{32}$$

Let $F(t, x) = \phi^2(t, x, y(t, x))$, from

$$J^3\left(\frac{\phi}{x}\right)\bigg|_{\vec{z}^*} = 0, \qquad D^2\left(\frac{\phi}{x}\right)\bigg|_{\vec{z}^*} = 0, \tag{33}$$

one can respectively prove

$$\frac{\partial F}{\partial x}\bigg|_{\vec{z}^*} = 0, \qquad \frac{\partial F}{\partial t}\bigg|_{\vec{z}^*} = 0. \tag{34}$$

Then the Taylor expansion of $F(x, t)$ in the neighborhood of $\vec{w}^*$ can be expressed as

$$F(x, t) = \tfrac{1}{2}A(x - x^*)^2 + B(x - x^*)(t - t^*) + \tfrac{1}{2}C(t - t^*)^2 + \cdots, \tag{35}$$

where

$$A = \frac{\partial^2 F}{\partial x^2}, \qquad B = \frac{\partial^2 F}{\partial x \partial t}, \qquad C = \frac{\partial^2 F}{\partial t^2}. \tag{36}$$

Note that

$$F(t^*, x^*) = \phi^2(t^*, x^*, y(t^*, x^*)), \tag{37}$$

we have

$$A\left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right)^2 + 2B\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*} + C = 0 \qquad \left(\text{or} \quad C\left(\left.\frac{\mathrm{d}t}{\mathrm{d}x}\right|_{\vec{w}^*}\right)^2 + 2B\left.\frac{\mathrm{d}t}{\mathrm{d}x}\right|_{\vec{w}^*} + A = 0\right). \tag{38}$$

(1) If $A \neq 0$, $B^2 - 4AC > 0$, then

$$\left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right)_1 = \frac{-B + \sqrt{B^2 - AC}}{A}, \qquad \left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right)_2 = \frac{-B - \sqrt{B^2 - AC}}{A}, \tag{39}$$

which simply represent the intersection of two vortex lines.

(2) If $A \neq 0$, $B^2 - 4AC = 0$, then

$$\left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right)_1 = \left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right)_2 = -\frac{B}{A}, \tag{40}$$

which could be either one vortex line splits into two vortex lines, or two vortex lines merge into one vortex line, with a speed of $v = -\frac{B}{A}$. The same as before, the winding number $\beta\eta$ is conserved in the process, i.e.

$$\beta_1\eta_1 + \beta_2\eta_2 = \beta\eta. \tag{41}$$

(3) If $A = 0$, $B \neq 0$, then

$$\left(\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\vec{w}^*}\right) = -\frac{C}{2B}, \tag{42}$$

which simply describes the motion of one vortex line.

4. The reduced dynamic equation and conserved dynamic quantity on stable vortex lines

In section 2, we have obtained the dynamic equation of the polarized condensed atoms (7):

$$\begin{aligned}-\hbar\varepsilon_{ab}\phi^a\frac{\partial}{\partial t}\phi^b = &-\frac{\hbar^2}{2m}\phi^a\nabla^2\phi^a + \frac{\hbar}{m}(\alpha\vec{E}\times\vec{B})\cdot\varepsilon_{ab}\phi^a\nabla\phi^b \\ &+ \left[\frac{1}{2m}(\alpha\vec{E}\times\vec{B})^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + V\right]\|\phi\|^2.\end{aligned} \tag{43}$$

Note that

$$\frac{1}{\|\phi\|^2}\phi^a\nabla^2\phi^a = \frac{\nabla^2\|\phi\|}{\|\phi\|} - \left(\nabla\frac{\phi^a}{\|\phi\|}\right)^2, \tag{44}$$

we then have

$$\begin{aligned}\hbar\frac{1}{\|\phi\|^2}\varepsilon_{ab}\phi^a\frac{\partial}{\partial t}\phi^b = &-\frac{\hbar^2}{2m}\left(\nabla\frac{\phi^a}{\|\phi\|}\right)^2 - \frac{\hbar}{m}(\alpha\vec{E}\times\vec{B})\cdot\frac{1}{\|\phi\|^2}\varepsilon_{ab}\phi^a\nabla\phi^b \\ &- \frac{1}{2m}(\alpha\vec{E}\times\vec{B})^2 + \frac{\alpha}{2}E^2 - \frac{g}{2}\|\phi\|^2 - U - V,\end{aligned} \tag{45}$$

where

$$U = -\frac{\hbar^2}{2m}\frac{\nabla^2\|\phi\|}{\|\phi\|} \tag{46}$$

is the Bohm quantum potential [16].

Differentiate equation (45) with respect to coordinates. Using (10), it is easy to prove

$$-\frac{\hbar^2}{2m}\nabla\left(\nabla\frac{\phi^a}{\|\phi\|}\right)^2 = m\vec{u}\times(\nabla\times\vec{u}) - \nabla\left(\frac{1}{2}mu^2\right) + \nabla[(\alpha\vec{E}\times\vec{B})\cdot\vec{u}] + \frac{1}{2m}\nabla(\alpha\vec{E}\times\vec{B})^2, \tag{47}$$

$$\hbar\nabla\left(\frac{1}{\|\phi\|^2}\varepsilon_{ab}\phi^a\frac{\partial}{\partial t}\phi^b\right) = m\frac{\partial\vec{u}}{\partial t} - m\frac{\partial\vec{x}}{\partial t}\times(\nabla\times\vec{u}), \tag{48}$$

then we obtain the reduced dynamical equation

$$m\frac{\partial\vec{u}}{\partial t} = m\left(\vec{u}+\frac{\partial\vec{x}}{\partial t}\right)\times(\nabla\times\vec{u}) - \nabla\left(\frac{1}{2}mu^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + U + V\right). \tag{49}$$

For stable vortex lines, $\frac{\partial\vec{u}}{\partial t} = 0$, we have

$$m\left(\vec{u}+\frac{\partial\vec{x}}{\partial t}\right)\times(\nabla\times\vec{u}) = \nabla\left(\frac{1}{2}mu^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + U + V\right), \tag{50}$$

then

$$\nabla\left(\frac{1}{2}mu^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + U + V\right)\cdot(\nabla\times\vec{u}) = 0. \tag{51}$$

Just like the situation in fluid mechanics, on those vortex lines

$$(\nabla\times\vec{u})^i \propto \mathrm{d}x^i. \tag{52}$$

Equation (51) becomes

$$\partial_i\left(\frac{1}{2}mu^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + U + V\right)\mathrm{d}x^i = 0, \tag{53}$$

thus

$$\frac{1}{2}mu^2 - \frac{\alpha}{2}E^2 + \frac{g}{2}\|\phi\|^2 + U + V = \text{const}. \tag{54}$$

It is a conserved dynamics quantity on the stable vortex lines.

5. Conclusions

The use of ϕ-mapping topological theory has clarified the issue on the generation of vortices in quantum Röntgen effect. The explicit expression of curl $\vec{u}$ is given, from which we get the vorticity of the velocity field and the magnetic flux of the monopoles; both of them are quantized by the product of the Hopf indices and the Brouwer degrees. The generating, annihilating, splitting and merging of vortex lines are discussed in detail, and the speeds of these processes are estimated, which prove handy when experiments are concerned. The reduced dynamic equation of the condensate is derived, from which we get a conserved dynamic quantity on the stable vortex lines.

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